

**Investigate Properties of Radicals Using a CAS (Computer Algebra System) GIFT  
Module 1**

1. a) Type into your CAS and write the answer:  $\sqrt{2} + \sqrt{3}$  \_\_\_\_\_  
b) Is  $\sqrt{2} + \sqrt{3} = \sqrt{5}$  ? \_\_\_\_\_ Explain:

2. a) Type into your CAS and write the answer:  $\sqrt{4} + \sqrt{9}$  \_\_\_\_\_  
b) Is  $\sqrt{4} + \sqrt{9} = \sqrt{13}$  ? \_\_\_\_\_ Explain:

3. Type each of the following into your CAS and write your answer:

- a)  $2\sqrt{3} + 5\sqrt{3} =$  \_\_\_\_\_ b)  $2\sqrt{7} + 4\sqrt{7} =$  \_\_\_\_\_  
c)  $3\sqrt{5} + \sqrt{5} =$  \_\_\_\_\_ d)  $\sqrt{6} + \sqrt{6} =$  \_\_\_\_\_ e)  $\sqrt{3} + \sqrt{5} =$  \_\_\_\_\_

4. Look at the input and output in exercises 1 – 3 above. Based on your observations:  
When CAN you add expressions involving radicals?  
When can you NOT add expressions involving radicals?  
Write some generalizations here.

5. a) Type into your CAS and write the answer:  $\sqrt{5} - \sqrt{3}$  \_\_\_\_\_  
b) Is  $\sqrt{5} - \sqrt{3} = \sqrt{2}$  ? \_\_\_\_\_ Explain:

6. a) Type into your CAS and write the answer:  $\sqrt{9} - \sqrt{4}$  \_\_\_\_\_  
b) Is  $\sqrt{9} - \sqrt{4} = \sqrt{5}$  ? \_\_\_\_\_ Explain:

7. Type each of the following into your CAS and write your answer:

- a)  $7\sqrt{6} - 2\sqrt{6} =$  \_\_\_\_\_ b)  $8\sqrt{3} - 2\sqrt{3} =$  \_\_\_\_\_  
c)  $4\sqrt{5} - \sqrt{5} =$  \_\_\_\_\_ d)  $2\sqrt{7} - 5\sqrt{7} =$  \_\_\_\_\_ e)  $\sqrt{7} - \sqrt{5} =$  \_\_\_\_\_

8. Look at the input and output in exercises 5 – 7 above. Based on your observations:  
When CAN you subtract expressions involving radicals?  
When can you NOT subtract expressions involving radicals?  
Write some generalizations here.

9. Type each of the following into your CAS and write your answer:

a)  $\sqrt{2} \cdot \sqrt{3} = \underline{\hspace{2cm}}$     b)  $\sqrt{3} \cdot \sqrt{5} = \underline{\hspace{2cm}}$     c)  $\sqrt{5} \cdot \sqrt{6} = \underline{\hspace{2cm}}$

d) Is  $\sqrt{7} \cdot \sqrt{3} = \sqrt{21}$ ?  $\underline{\hspace{2cm}}$  Explain.

10. Based on the 4 examples in exercise 9, can you make any generalization? Write your conclusions here.

11. Type each of the following into your CAS and write your answer:

a)  $\frac{\sqrt{6}}{\sqrt{2}} = \underline{\hspace{2cm}}$     b)  $\frac{\sqrt{35}}{\sqrt{5}} = \underline{\hspace{2cm}}$     c)  $\frac{\sqrt{30}}{\sqrt{6}} = \underline{\hspace{2cm}}$

d) Is  $\frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3}$ ?  $\underline{\hspace{2cm}}$  Explain.

12. Based on the 4 examples in exercise 11, can you make any generalization? Write your conclusions here.

13. It appears that sometimes you CAN add or subtract radical expressions and sometimes you can't. Make sure that you know when you CAN and when you CANNOT.

14. Discuss when you can multiply or divide radical expressions.

15 – 30. For each of the following, first simplify, if possible, withOUT using CAS. Then check your answer using CAS. Put a check mark or a ☺ if you got it right, an X if you got it wrong.

Expression	Your simplified form	CAS simplified form	☺ or X?
15. $\sqrt{5} + \sqrt{6}$	_____	_____	_____
16. $\sqrt{5} \cdot \sqrt{6}$	_____	_____	_____
17. $\sqrt{7} - \sqrt{5}$	_____	_____	_____
18. $\sqrt{7} \cdot \sqrt{5}$	_____	_____	_____
19. $\frac{\sqrt{10}}{\sqrt{2}}$	_____	_____	_____
20. $4\sqrt{5} + 3\sqrt{5}$	_____	_____	_____
21. $6\sqrt{10} - 8\sqrt{10}$	_____	_____	_____
22. $\sqrt{10} + \sqrt{3}$	_____	_____	_____
23. $\sqrt{10} \cdot \sqrt{3}$	_____	_____	_____
24. $\sqrt{10} - \sqrt{3}$	_____	_____	_____
25. $\sqrt{6} + \sqrt{6}$	_____	_____	_____
26. $\sqrt{6} \cdot \sqrt{6}$	_____	_____	_____
27. $\sqrt{6} - \sqrt{6}$	_____	_____	_____
28. $\frac{\sqrt{24}}{\sqrt{8}}$	_____	_____	_____
29. $5\sqrt{2} - \sqrt{2}$	_____	_____	_____
30. $\sqrt{11} + 5\sqrt{11}$	_____	_____	_____

**Module 2**

## Class Starter

1 – 4: a) Simplify, if possible, withOUT CAS. b) Check WITH CAS c) ☺ or X?

	No CAS	Yes CAS	☺ or X?
1. $\sqrt{10} - \sqrt{6}$	_____	_____	_____
2. $\sqrt{5} \cdot \sqrt{7}$	_____	_____	_____
3. $4\sqrt{2} + 5\sqrt{2}$	_____	_____	_____
4. $\frac{\sqrt{22}}{\sqrt{2}}$	_____	_____	_____

**Exploration**

5 – 8. Type each of the following into your CAS and write your answer:

5.  $(\sqrt{7})^2 =$  \_\_\_\_\_ 6.  $(\sqrt{18})^2 =$  \_\_\_\_\_ 7.  $\sqrt{15^2} =$  \_\_\_\_\_ 8.  $\sqrt{325^2} =$  \_\_\_\_\_

9 – 12. . Look at the input and output in exercises 5 – 8 above. Based on your observations, discuss in your groups and try to make a generalization. Then simplify the following withOUT a CAS, then check with a CAS (in that order).

	No CAS	Yes CAS	☺ or X?
9. $(\sqrt{6})^2 =$	_____	_____	_____
10. $\sqrt{14^2} =$	_____	_____	_____
11. $(\sqrt{232})^2 =$	_____	_____	_____
12. $\sqrt{1047^2} =$	_____	_____	_____

**Exploration**

13 – 16. Type each of the following into your CAS and write your answer:

13.  $5\sqrt{3} \cdot 4\sqrt{2} =$  \_\_\_\_\_ 14.  $(-6\sqrt{7})(5\sqrt{3}) =$  \_\_\_\_\_

15.  $-3\sqrt{5} \cdot -2\sqrt{6} =$  \_\_\_\_\_ 16.  $(3\sqrt{5})(3\sqrt{5}) =$  \_\_\_\_\_

17 – 20. Look at the input and output in exercises 13 – 16. Based on your observations, discuss in your groups and try to make a generalization. Then simplify the following withOUT a CAS, then check with a CAS (in that order).

	No CAS	Yes CAS	☺ or X?
17. $4\sqrt{7} \cdot 2\sqrt{3} =$	_____	_____	_____
18. $(-5\sqrt{6})(3\sqrt{7}) =$	_____	_____	_____
19. $-6\sqrt{2} \cdot -\sqrt{5} =$	_____	_____	_____
20. $(4\sqrt{3})(4\sqrt{3}) =$	_____	_____	_____

### Exploration

21 – 24. Type each of the following into your CAS and write your answer:

21.  $(2\sqrt{3})^2 =$  \_\_\_\_\_ 22.  $(5\sqrt{2})^2 =$  \_\_\_\_\_ 23.  $(3\sqrt{6})^2 =$  \_\_\_\_\_ 24.  $(4\sqrt{5})^2 =$  \_\_\_\_\_

25 – 28. Look at the input and output in exercises 13 – 16. Based on your observations, discuss in your groups and try to make a generalization. Then simplify the following withOUT a CAS, then check with a CAS (in that order).

	No CAS	Yes CAS	☺ or X?
25. $(2\sqrt{5})^2 =$	_____	_____	_____
26. $(3\sqrt{7})^2 =$	_____	_____	_____
27. $(4\sqrt{2})^2 =$	_____	_____	_____
28. $(5\sqrt{3})^2 =$	_____	_____	_____

**29 – 41. SELF CHECK.** Procedure. Simplify the expression, if it is possible. Then check your answer with CAS. Do one exercise at a time. Learn!!! Show all work on this paper.

	No CAS	Yes CAS	☺ or X?
29. $4\sqrt{6} + \sqrt{6} =$	_____	_____	_____
30. $\sqrt{15} - \sqrt{5} =$	_____	_____	_____
31. $(\sqrt{3})(\sqrt{7}) =$	_____	_____	_____
32. $\frac{\sqrt{6}}{\sqrt{3}} =$	_____	_____	_____
33. $\sqrt{7} + \sqrt{7} =$	_____	_____	_____
34. $\sqrt{7} \cdot \sqrt{7} =$	_____	_____	_____
35. $(\sqrt{23})^2 =$	_____	_____	_____
36. $3\sqrt{5} \cdot 4\sqrt{2} =$	_____	_____	_____
37. $(2\sqrt{3})^2 =$	_____	_____	_____
38. $\sqrt{5} \cdot \sqrt{3} =$	_____	_____	_____
39. $\sqrt{6} \cdot 3 =$	_____	_____	_____
40. $2\sqrt{7} + 3\sqrt{5} =$	_____	_____	_____
41. $(2\sqrt{7})(3\sqrt{5}) =$	_____	_____	_____

### Module 3 Exploration

1 – 12. Type each of the following into your CAS and write your answer:

- Simplify:  $\sqrt{8} =$  \_\_\_\_\_
- Simplify:  $\sqrt{18} =$  \_\_\_\_\_
- Simplify:  $\sqrt{32} =$  \_\_\_\_\_
- Simplify:  $\sqrt{50} =$  \_\_\_\_\_
- Do you see a pattern yet? Discuss.

6. Simplify:  $\sqrt{12} =$  \_\_\_\_\_      7. Simplify:  $\sqrt{30} =$  \_\_\_\_\_
8. Simplify:  $\sqrt{20} =$  \_\_\_\_\_      9. Simplify:  $\sqrt{63} =$  \_\_\_\_\_
10. Simplify:  $\sqrt{15} =$  \_\_\_\_\_      11. Simplify:  $\sqrt{40} =$  \_\_\_\_\_

12. Now do you see a pattern? It may be a bit tough, but discuss in your groups. Brainstorm.

## CLASS DISCUSSION AND REFLECTION

### HOW TO SIMPLIFY RADICAL EXPRESSIONS ALGEBRAICALLY

What is meant when we say that a radical expression is in simplest form, that is, what are the requirements?

## EXERCISES

For problems 1 – 4, and 6 – 11, on your paper, show algebraically how the CAS simplified the radical expressions, or why the expression was already in simplest form. Number your solutions with the same numbers, 1 – 4 and 6 – 11.

13 – 24. On you paper, simplify each radical expression, if possible. Then check that you are correct with the CAS. Make any corrections as needed.

Do and check one problem before continuing to the next one. Learn from any mistakes. SIMPLIFY. Show each step.

- |                  |                 |                  |                 |
|------------------|-----------------|------------------|-----------------|
| 13. $\sqrt{45}$  | 14. $\sqrt{24}$ | 15. $\sqrt{75}$  | 16. $\sqrt{27}$ |
| 17. $\sqrt{10}$  | 18. $\sqrt{28}$ | 19. $\sqrt{500}$ | 20. $\sqrt{98}$ |
| 21. $\sqrt{162}$ | 22. $\sqrt{26}$ | 23. $\sqrt{48}$  | 24. $\sqrt{72}$ |

**Exploration**

25 – 27. Type each of the following into your CAS and write your answer:

25.  $\sqrt{8} + \sqrt{18} =$  \_\_\_\_\_ 26.  $\sqrt{5} + \sqrt{20} =$  \_\_\_\_\_ 27.  $\sqrt{63} - \sqrt{28} =$  \_\_\_\_\_

28. In your groups discuss how you can algebraically obtain the answers that the CAS did for exercises 25 to 27. Show the steps on your paper for EACH problem.

**29 - 32. SELF CHECK.** Procedure. Simplify the expression, if it is possible. Then check your answer with CAS. Do one exercise at a time. Learn!!! Show all work on your paper.

	No CAS	Yes CAS	☺ or X?
29. $\sqrt{12} + \sqrt{27} =$	_____	_____	_____
30. $\sqrt{50} + \sqrt{8} =$	_____	_____	_____
31. $\sqrt{54} - \sqrt{24} =$	_____	_____	_____
32. $\sqrt{500} - \sqrt{20} =$	_____	_____	_____

**Module 4 Summary Exercises (so far)**

Copy the problems onto your paper and simplify algebraically, if possible.

After each group of 4 problems, use your CAS to check and make any corrections.

LEARN from the CAS!!! And... enjoy. NOTE: numbers 10 and 23 cannot be checked with CAS.

**1 – 24. SIMPLIFY:**

1.  $\sqrt{12}$                       2.  $\sqrt{5} \cdot \sqrt{6}$                       3.  $(\sqrt{317})^2$                       4.  $\sqrt{3} + \sqrt{3}$

Now check 1 to 4.

5.  $\sqrt{3} \cdot \sqrt{3}$                       6.  $\sqrt{28}$                       7.  $(4\sqrt{5})^2$                       8.  $4\sqrt{3} + 5\sqrt{2}$

Now check 5 to 8.

9.  $4\sqrt{3} \cdot 5\sqrt{2}$                       10.  $\sqrt{3}(\sqrt{7} + 4)$                       11.  $\sqrt{45} + \sqrt{20}$                       12.  $-2\sqrt{7} \cdot \sqrt{7}$

Now check 9 to 12.

13.  $-2\sqrt{7} + \sqrt{7}$                       14.  $\sqrt{34^2}$                       15.  $\sqrt{50} - \sqrt{18}$                       16.  $\sqrt{10} - \sqrt{10}$

Now check 13 to 16.

17.  $\sqrt{10} \cdot -\sqrt{10}$                       18.  $(3\sqrt{6})^2$                       19.  $\sqrt{300}$                       20.  $\frac{\sqrt{30}}{\sqrt{6}}$

Now check 17 to 20.

21.  $\sqrt{6} - \sqrt{5}$                       22.  $\sqrt{32}$                       23.  $\sqrt{6}(\sqrt{7} - \sqrt{5})$                       24.  $\frac{-2\sqrt{7}}{\sqrt{7}}$